OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) 4753/01

Methods for Advanced Mathematics (C3)

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:
- Printed answer book 4753/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Monday 20 June 2011
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The printed answer book consists of **16** pages. The question paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

Section A (36 marks)

1 Solve the equation $|2x - 1| = |x|$. **[4]**

2 Given that $f(x) = 2\ln x$ and $g(x) = e^x$, find the composite function $gf(x)$, expressing your answer as simply as possible. **[3]**

3 **(i)** Differentiate $\dfrac{\ln x}{x^2}$, simplifying your answer. **[4]**

(ii) Using integration by parts, show that $\displaystyle\int \frac{\ln x}{x^2}\,dx = -\frac{1}{x}(1 + \ln x) + c$. **[4]**

4 The height h metres of a tree after t years is modelled by the equation

$$h = a - be^{-kt},$$

where a, b and k are positive constants.

(i) Given that the long-term height of the tree is 10.5 metres, and the initial height is 0.5 metres, find the values of a and b. **[3]**

(ii) Given also that the tree grows to a height of 6 metres in 8 years, find the value of k, giving your answer correct to 2 decimal places. **[3]**

5 Given that $y = x^2\sqrt{1 + 4x}$, show that $\dfrac{dy}{dx} = \dfrac{2x(5x + 1)}{\sqrt{1 + 4x}}$. **[5]**

6 A curve is defined by the equation $\sin 2x + \cos y = \sqrt{3}$.

(i) Verify that the point $P\left(\frac{1}{6}\pi, \frac{1}{6}\pi\right)$ lies on the curve. **[1]**

(ii) Find $\dfrac{dy}{dx}$ in terms of x and y.

Hence find the gradient of the curve at the point P. **[5]**

7 **(i)** Multiply out $(3^n + 1)(3^n - 1)$. **[1]**

(ii) Hence prove that if n is a positive integer then $3^{2n} - 1$ is divisible by 8. **[3]**

Section B (36 marks)

8

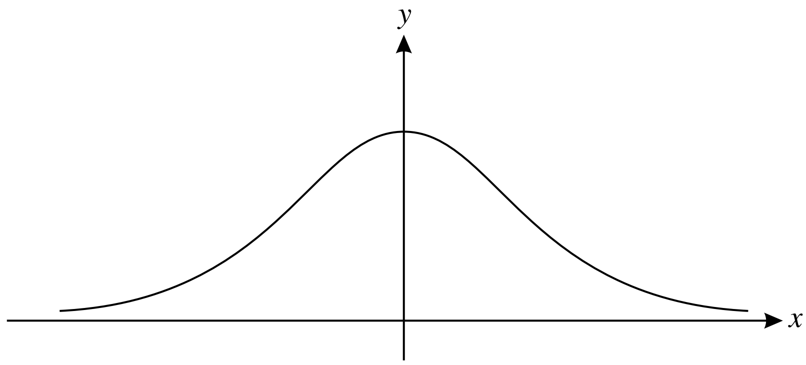

Fig. 8

Fig. 8 shows the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{1}{\mathrm{e}^x + \mathrm{e}^{-x} + 2}$.

(i) Show algebraically that $\mathrm{f}(x)$ is an even function, and state how this property relates to the curve $y = \mathrm{f}(x)$. **[3]**

(ii) Find $\mathrm{f}'(x)$. **[3]**

(iii) Show that $\mathrm{f}(x) = \dfrac{\mathrm{e}^x}{(\mathrm{e}^x + 1)^2}$. **[2]**

(iv) Hence, using the substitution $u = \mathrm{e}^x + 1$, or otherwise, find the exact area enclosed by the curve $y = \mathrm{f}(x)$, the x-axis, and the lines $x = 0$ and $x = 1$. **[5]**

(v) Show that there is only one point of intersection of the curves $y = \mathrm{f}(x)$ and $y = \frac{1}{4}\mathrm{e}^x$, and find its coordinates. **[5]**

[Question 9 is printed overleaf.]

9 Fig. 9 shows the curve $y = \mathrm{f}(x)$. The endpoints of the curve are P $(-\pi, 1)$ and Q $(\pi, 3)$, and $\mathrm{f}(x) = a + \sin bx$, where a and b are constants.

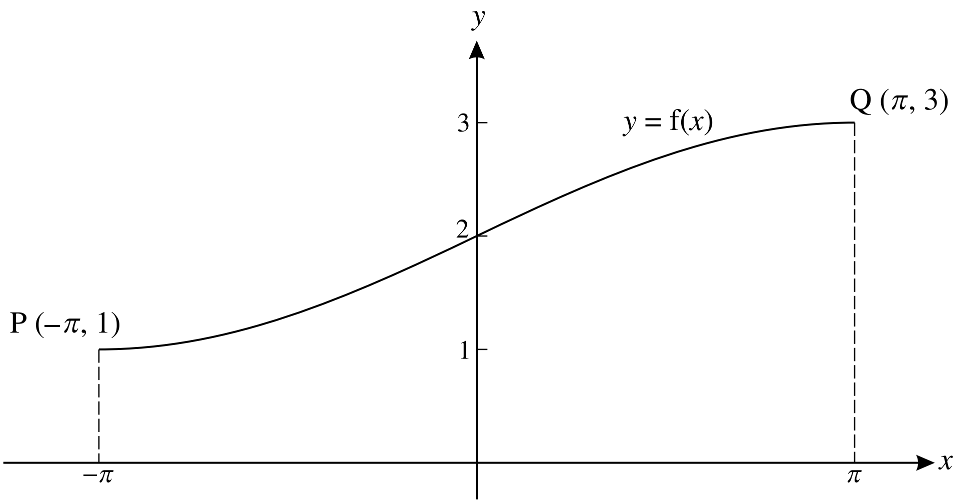

Fig. 9

(i) Using Fig. 9, show that $a = 2$ and $b = \frac{1}{2}$. **[3]**

(ii) Find the gradient of the curve $y = \mathrm{f}(x)$ at the point $(0, 2)$.

Show that there is no point on the curve at which the gradient is greater than this. **[5]**

(iii) Find $\mathrm{f}^{-1}(x)$, and state its domain and range.

Write down the gradient of $y = \mathrm{f}^{-1}(x)$ at the point $(2, 0)$. **[6]**

(iv) Find the area enclosed by the curve $y = \mathrm{f}(x)$, the x-axis, the y-axis and the line $x = \pi$. **[4]**

OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) **4753/01**

Methods for Advanced Mathematics (C3)

PRINTED ANSWER BOOK

Candidates answer on this printed answer book.

OCR supplied materials:
- Question paper 4753/01 (inserted)
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Monday 20 June 2011
Morning

Duration: 1 hour 30 minutes

Candidate forename		Candidate surname	

Centre number						Candidate number				

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The printed answer book consists of **16** pages. The question paper consists of **4** pages. Any blank pages are indicated.

Turn over

Section A (36 marks)

1

2

2	(continued)
3 (i)	

3 (ii)

4 (i)

4 (ii)

5

6 (i)	
6 (ii)	

7 (i)

7 (ii)

Section B (36 marks)

8 (i)	
8 (ii)	
	(answer space continued overleaf)

8 (ii)	(continued)
8 (iii)	

8 (iv)

8 (v)

9 (i)

9 (ii)	

9 (iii)

9 (iv)	

RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE

Unit **4753:** Methods for Advanced Mathematics

Mark Scheme for June 2011

Oxford Cambridge and RSA Examinations

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by Examiners. It does not indicate the details of the discussions which took place at an Examiners' meeting before marking commenced.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the Report on the Examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Any enquiries about publications should be addressed to:

OCR Publications
PO Box 5050
Annesley
NOTTINGHAM
NG15 0DL

Telephone: 0870 770 6622
Facsimile: 01223 552610
E-mail: publications@ocr.org.uk

Marking instructions for GCE Mathematics (MEI): Pure strand

1. You are advised to work through the paper yourself first. Ensure you familiarise yourself with the mark scheme before you tackle the practice scripts.

2. You will be required to mark ten practice scripts. This will help you to understand the mark scheme and will not be used to assess the quality of your marking. Mark the scripts yourself first, using the annotations. Turn on the comments box and make sure you understand the comments. You must also look at the definitive marks to check your marking. If you are unsure why the marks for the practice scripts have been awarded in the way they have, please contact your Team Leader.

3. When you are confident with the mark scheme, mark the ten standardisation scripts. Your Team Leader will give you feedback on these scripts and approve you for marking. (If your marking is not of an acceptable standard your Team Leader will give you advice and you will be required to do further work. You will only be approved for marking if your Team Leader is confident that you will be able to mark candidate scripts to an acceptable standard.)

4. Mark strictly to the mark scheme. If in doubt, consult your Team Leader using the messaging system within *scoris*, by email or by telephone. Your Team Leader will be monitoring your marking and giving you feedback throughout the marking period.

   An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

   Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

5. The following types of marks are available.

   M
   A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

   A
   Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

   B
   Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

6. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

7. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

   Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

8. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

9. **Rules for crossed out and/or replaced work**

   If work is crossed out and not replaced, examiners should mark the crossed out work if it is legible.

   If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

   If two or more attempts are made at a question, and just one is not crossed out, examiners should ignore the crossed out work and mark the work that is not crossed out.

   If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

   NB Follow these maths-specific instructions rather than those in the assessor handbook.

10. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

    Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

11. Annotations should be used whenever appropriate during your marking.

    The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

    For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

12. For answers scoring no marks, you must either award NR (no response) or 0, as follows:

    Award NR (no response) if:
    - Nothing is written at all in the answer space
    - There is a comment which does not in any way relate to the question being asked ("can't do", "don't know", etc.)
    - There is any sort of mark that is not an attempt at the question (a dash, a question mark, etc.)

    The hash key [#] on your keyboard will enter NR.

    Award 0 if:
    - There is an attempt that earns no credit. This could, for example, include the candidate copying all or some of the question, or any working that does not earn any marks, whether crossed out or not.

13. The following abbreviations may be used in this mark scheme.

M1	method mark (M2, etc, is also used)
A1	accuracy mark
B1	independent mark
E1	mark for explaining
U1	mark for correct units
G1	mark for a correct feature on a graph
M1 dep*	method mark dependent on a previous mark, indicated by *
cao	correct answer only
ft	follow through
isw	ignore subsequent working
oe	or equivalent
rot	rounded or truncated
sc	special case
soi	seen or implied
www	without wrong working

14. Annotating scripts. The following annotations are available:

    ✓and ✗
    BOD Benefit of doubt
    FT Follow through
    ISW Ignore subsequent working (after correct answer obtained)
    M0, M1 Method mark awarded 0, 1
    A0, A1 Accuracy mark awarded 0, 1
    B0, B1 Independent mark awarded 0,1
    SC Special case
    ^ Omission sign
    MR Misread
    Highlighting is also available to highlight any particular points on a script.

15. The comments box will be used by the Principal Examiner to explain his or her marking of the practice scripts for your information. Please refer to these comments when checking your practice scripts.

    Please do not type in the comments box yourself. Any questions or comments you have for your Team Leader should be communicated by the *scoris* messaging system, e-mail or by telephone.

16. Write a brief report on the performance of the candidates. Your Team Leader will tell you when this is required. The Assistant Examiner's Report Form (AERF) can be found on the Cambridge Assessment Support Portal. This should contain notes on particular strengths displayed, as well as common errors or weaknesses. Constructive criticisms of the question paper/mark scheme are also appreciated.

17. Link Additional Objects with work relating to a question to those questions (a chain link appears by the relevant question number) – see scoris assessor Quick Reference Guide page 19-20 for instructions as to how to do this – this guide is on the Cambridge Assessment Support Portal and new users may like to download it with a shortcut on your desktop so you can open it easily! For AOs containing just formulae or rough working not attributed to a question, tick at the top to indicate seen but not linked. When you submit the script, *scoris* asks you to confirm that you have looked at all the additional objects. Please ensure that you have checked all Additional Objects thoroughly.

18. The schedule of dates for the marking of this paper is displayed under 'OCR Subject Specific Details' on the Cambridge Assessment Support Portal. It is vitally important that you meet these requirements. If you experience problems that mean you may not be able to meet the deadline then you must contact your Team Leader without delay.

1 $\lvert 2x-1\rvert=\lvert x\rvert$ $\Rightarrow$ $2x - 1 = x$, $x = 1$ or $-(2x - 1) = x$, $x = 1/3$	M1A1 M1A1 [4]	www www, or $2x - 1 = -x$ must be exact for A1 (e.g. not 0.33, but allow $0.\dot{3}$) condone doing both equalities in one line e.g. $-x = 2x - 1 = x$, etc	allow unsupported answers or from graph or squaring $\Rightarrow 3x^2 - 4x + 1 = 0$ M1 $\Rightarrow (3x - 1)(x - 1) = 0$ M1 factorising, formula or comp. square $\Rightarrow x = 1, 1/3$ A1 A1 allow M1 for sign errors in factorisation -1 if more than two solutions offered, but isw inequalities
2 $\text{gf}(x) = e^{2\ln x}$ $= e^{\ln x^2}$ $= x^2$	M1 M1 A1 [3]	Forming gf(x) (soi)	Doing fg: $2\ln(e^x) = 2x$ SC1 Allow x^2 (but not $2x$) unsupported
3(i) $\dfrac{dy}{dx}=\dfrac{x^2.\frac{1}{x}-\ln x.2x}{x^4}$ $=\dfrac{x-2x\ln x}{x^4}$ $=\dfrac{1-2\ln x}{x^3}$	M1 B1 A1 A1 [4]	quotient rule with $u = \ln x$ and $v = x^2$ $d/dx (\ln x) = 1/x$ soi correct expression (o.e.) o.e. cao, mark final answer, but must have divided top and bottom by x	Consistent with their derivatives. $udv \pm vdu$ in the quotient rule is M0 Condone $\ln x.2x = \ln 2x^2$ for this A1 (provided $\ln x.2x$ is shown) e.g. $\dfrac{1}{x^3}-\dfrac{2\ln x}{x^3}$, $x^{-3}-2x^{-3}\ln x$
or $\dfrac{dy}{dx}=-2x^{-3}\ln x+x^{-2}(\frac{1}{x})$ $=-2x^{-3}\ln x+x^{-3}$	M1 B1 A1 A1 [4]	product rule with $u = x^{-2}$ and $v = \ln x$ $d/dx (\ln x) = 1/x$ soi correct expression o.e. cao, mark final answer, must simplify the $x^{-2}.(1/x)$ term.	or vice-versa
(ii) $\int\dfrac{\ln x}{x^2}dx$ let $u = \ln x$, $du/dx = 1/x$ $dv/dx = 1/x^2$, $v = -x^{-1}$ $=-\dfrac{1}{x}\ln x+\int\dfrac{1}{x}.\dfrac{1}{x}dx$ $=-\dfrac{1}{x}\ln x+\int\dfrac{1}{x^2}dx$ $=-\dfrac{1}{x}\ln x-\dfrac{1}{x}+c$ $=-\dfrac{1}{x}(\ln x+1)+c$ *	M1 A1 A1 A1 [4]	Integration by parts with $u = \ln x$, $du/dx = 1/x$, $dv/dx = 1/x^2$, $v = -x^{-1}$ must be correct, condone + c condone missing c NB **AG** must have c shown in final answer	Must be correct at this stage . Need to see $1/x^2$

4(i) $h = a - be^{-kt} \Rightarrow a = 10.5$ (their)$a - be^0 = 0.5$ $\Rightarrow b = 10$	B1 M1 A1cao [3]	a need not be substituted	
(ii) $h = 10.5 - 10e^{-kt}$ When $t = 8$, $h = 10.5 - 10e^{-8k} = 6$ $\Rightarrow 10e^{-8k} = 4.5$ $\Rightarrow -8k = \ln 0.45$ $\Rightarrow k = \ln 0.45/(-8) = 0.09981\ldots = 0.10$	M1 M1 A1 [3]	ft their a and b (even if made up) taking lns correctly on a correct re-arrangement - ft a, b if not eased cao (www) but allow 0.1	allow M1 for $a - be^{-8k} = 6$ allow a and b unsubstituted allow their 0.45 (or 4.5) to be negative
5 $y = x^2(1+4x)^{1/2}$ $\Rightarrow \frac{dy}{dx} = x^2.\frac{1}{2}(1+4x)^{-1/2}.4 + 2x(1+4x)^{1/2}$ $= 2x(1+4x)^{-1/2}(x+1+4x)$ $= \frac{2x(5x+1)}{\sqrt{1+4x}}$ *	M1 B1 A1 M1 A1 [5]	product rule with $u = x^2$, $v = \sqrt{(1+4x)}$ ½ $(\ldots)^{-1/2}$ soi correct expression factorising or combining fractions NB **AG**	consistent with their derivatives; condone wrong index in v used for M1 only (need not factor out the $2x$) must have evidence of $x + 1 + 4x$ oe or $2x(5x+1)(1+4x)^{-½}$ or $2x(5x+1)/(1+4x)^{½}$
6(i) $\sin(\pi/3) + \cos(\pi/6) = \sqrt{3}/2 + \sqrt{3}/2 = \sqrt{3}$	B1 [1]	must be exact, must show working	Not just $\sin(\pi/3) + \cos(\pi/6) = \sqrt{3}$, if substituting for y and solving for x (or vv) must evaluate $\sin \pi/3$ e.g. not $\arccos(\sqrt{3} - \sin \pi/3)$
(ii) $2\cos 2x - \sin y \frac{dy}{dx} = 0$ $\Rightarrow 2\cos 2x = \sin y \frac{dy}{dx}$ $\Rightarrow \frac{dy}{dx} = \frac{2\cos 2x}{\sin y}$ When $x = \pi/6$, $y = \pi/6$ $\Rightarrow \frac{dy}{dx} = \frac{2\cos \pi/3}{\sin \pi/6} = 2$	M1 A1 A1cao M1dep A1 [5]	Implicit differentiation correct expression substituting dep 1st M1 www	allow one error, but must have (±) $\sin y\, dy/dx$. Ignore $dy/dx = \ldots$ unless pursued. $2\cos 2x\, dx - \sin y\, dy = 0$ is M1A1 (could differentiate wrt y, get dx/dy, etc.) $\frac{-2\cos 2x}{-\sin y}$ is A0 or 30°
7 (i) $(3^n + 1)(3^n - 1) = (3^n)^2 - 1$ or $3^{2n} - 1$	B1 [1]	mark final answer	or $9^n - 1$; penalise 3^{n^2} if it looks like 3 to the power n^2.
(ii) 3^n is odd $\Rightarrow 3^n + 1$ and $3^n - 1$ both even As consecutive even nos, one must be divisible by 4, so product is divisible by 8.	M1 M1 A1 [3]	3^n is odd $\Rightarrow 3^n + 1$ and $3^n - 1$ both even completion	Induction: If true for n, $3^{2n} - 1 = 8k$, so $3^{2n} = 1 + 8k$, M1 $3^{2(n+1)} - 1 = 9\times(8k+1) - 1 = 72k + 8 = 8(9k+1)$ so div by 8. A1 When $n = 1$, $3^2 - 1 = 8$ div by 8, true A1(or similar with 9^n)

8(i) $f(-x)=\frac{1}{e^{-x}+e^{-(-x)}+2}$ $= f(x)$, [$\Rightarrow$ f is even *] Symmetrical about Oy	M1 A1 B1 [3]	substituting $-x$ for x in f(x) condone 'reflection in y-axis'	Can imply that $e^{-(-x)} = e^x$ from $f(-x)=\frac{1}{e^{-x}+e^{x}+2}$ Must mention axis
(ii) $f'(x) = -(e^x+e^{-x}+2)^{-2}(e^x-e^{-x})$ or $= \frac{(e^x+e^{-x}+2).0-(e^x-e^{-x})}{(e^x+e^{-x}+2)^2}$ $= \frac{(e^{-x}-e^x)}{(e^x+e^{-x}+2)^2}$	B1 M1 A1 [3]	d/dx (e^x) = e^x and d/dx(e^{-x}) = $-e^{-x}$ soi chain or quotient rule condone missing bracket on top if correct thereafter o.e. mark final answer	. If differentiating $\frac{e^x}{(e^x+1)^2}$ withhold A1 (unless result in (iii) proved here) e.g. $\frac{1}{(e^x+e^{-x}+2)^2}\times(e^{-x}-e^x)$
(iii) $f(x)=\frac{e^x}{e^{2x}+1+2e^x}$ $=\frac{e^x}{(e^x+1)^2}$ *	M1 A1 [2]	× top and bottom by e^x (correctly) condone e^{x^2} for M1 but not A1 **NB AG**	or $\frac{e^x}{(e^x+1)^2}=\frac{e^x}{e^{2x}+2e^x+1}$ M1, $=\frac{1}{e^x+e^{-x}+2}$ A1 condone no $e^{2x} = (e^x)^2$,for both M1 and A1
(iv) $A=\int_0^1 \frac{e^x}{(e^x+1)^2}\,dx$ let $u = e^x + 1$, du = e^x dx when x = 0, u = 2; when x = 1, u = e + 1 $\Rightarrow$ $A=\int_2^{1+e}\frac{1}{u^2}\,du$ $=\left[-\frac{1}{u}\right]_2^{1+e}$ $=-\frac{1}{1+e}+\frac{1}{2}=\frac{1}{2}-\frac{1}{1+e}$	B1 M1 A1 M1 A1cao [5]	correct integral and limits $\int\frac{1}{u^2}(du)$ $\left[-\frac{1}{u}\right]$ substituting correct limits (dep 1st M1 and integration) o.e. mark final answer. Must be exact Don't allow e^1.	condone no dx, must use $f(x)=\frac{e^x}{(e^x+1)^2}$. Limits may be implied by subsequent work. If 0.231.. unsupported, allow 1st B1 only or by inspection $\left[\frac{k}{e^x+1}\right]$ M1 $\left[-\frac{1}{e^x+1}\right]$ A1 upper–lower; 2 and 1+e (or 3.7..)for u, or 0 and 1 for x if substituted back (correctly) e.g. $\frac{e-1}{2(1+e)}$. Can isw 0.231, which may be used as evidence of M1. Can isw numerical ans (e.g. 0.231) but not algebraic errors
(v) Curves intersect when $f(x)=\frac{1}{4}e^x$ $\Rightarrow$ $(e^x + 1)^2 = 4$ $\Rightarrow$ $e^x = 1$ or -3 so as $e^x > 0$, only one solution $e^x = 1$ $\Rightarrow x = 0$ when $x = 0$, y = ¼	M1 M1 A1 B1 B1 [5]	soi or equivalent quadratic – must be correct getting $e^x = 1$ and discounting other sol^n x = 0 www (for this value) y = ¼ www (for the x value)	$\frac{e^x}{(e^x+1)^2}$ or $\frac{1}{e^x+e^{-x}+2}=\frac{1}{4}e^x$ With e^{2x} or $(e^x)^2$, condone e^{x^2}, e^0 www e.g. $e^x = -1$ [or $e^x + 1 = -2$] not possible www unless verified Do not allow unsupported. A sketch is not sufficient

9(i) When $x = 0$, $f(x) = a = 2$* When $x = \pi$, $f(\pi) = 2 + \sin b\pi = 3$ $\Rightarrow$ $\sin b\pi = 1$ $\Rightarrow$ $b\pi = ½\pi$, so $b = ½$ * *or* $1 = a + \sin(-\pi b)$ $(= a - \sin \pi b)$ $3 = a + \sin(\pi b)$ $\Rightarrow$ $2 = 2\sin \pi b$, $\sin \pi b = 1$, $\pi b = \pi/2$, $b = ½$ $\Rightarrow$ $3 = a + 1$ or $1 = a - 1 \Rightarrow a = 2$ (oe for b)	B1 M1 A1 [3]	NB **AG** 'a is the y-intercept' not enough but allow verification ($2+\sin 0 = 2$) or when $x = -\pi$, $f(-\pi) = 2 + \sin(-b\pi) = 1$ $\Rightarrow \sin(-b\pi) = -1$ condone using degrees $\Rightarrow -b\pi = -½\pi$, $b = ½$ **NB AG** M1 for both points substituted A1 solving for b or a A1 substituting to get a (or b)	or equiv transformation arguments : e.g. 'curve is shifted up 2 so $a = 2$'. e.g. period of sine curve is 4π, or stretched by sf. 2 in x-direction (not squeezed or squashed by ½) $\Rightarrow b = ½$ If verified allow M1A0 If $y = 2 + \sin ½x$ verified at two points, SC2 A sequence of sketches starting from $y = \sin x$ showing clearly the translation and the stretch (in either order) can earn full marks
(ii) $f'(x) = ½\cos ½x$ $\Rightarrow$ $f'(0) = ½$ Maximum value of $\cos ½x$ is 1 $\Rightarrow$ max value of gradient is ½	M1 A1 A1 M1 A1 [5]	$\pm k\cos ½x$ cao www or $f''(x) = -¼\sin ½x$ $f''(x) = 0 \Rightarrow x = 0$, so max val of $f'(x)$ is ½	
(iii) $y = 2 + \sin ½x$ $x \leftrightarrow y$ $x = 2 + \sin ½y$ $\Rightarrow$ $x - 2 = \sin ½y$ $\Rightarrow$ $\arcsin(x-2) = ½y$ $\Rightarrow$ $y = f^{-1}(x) = 2\arcsin(x-2)$ Domain $1 \le x \le 3$ Range $-\pi \le y \le \pi$ Gradient at (2, 0) is 2	M1 A1 A1 B1 B1 B1ft [6]	Attempt to invert formula or $\arcsin(y-2) = ½x$ must be $y = \ldots$ or $f^{-1}(x) = \ldots$ or [1, 3] or $[-\pi, \pi]$ or $-\pi \le f^{-1}(x) \le \pi$ ft their answer in (ii) (except ±1) 1/their ½	viz solve for x in terms of y or vice-versa – one step enough condone use of a and b in inverse function, e.g. $[\arcsin(x-a)]/b$ or $\sin^{-1}(y-2)$ condone no bracket for 1st A1 only or $2\sin^{-1}(x-2)$, condone $f'(x)$, must have bracket in final ans but not $1 \le y \le 3$ but not $-\pi \le x \le \pi$. Penalise <'s (or '1 to 3','$-\pi$ to π') once only or by differentiating $\arcsin(x-2)$ or implicitly
(iv) $A = \int_0^{\pi}(2+\sin\frac{1}{2}x)\,dx$ $= \left[2x - 2\cos\frac{1}{2}x\right]_0^{\pi}$ $= 2\pi - (-2)$ $= 2\pi + 2$ $(= 8.2831\ldots)$	M1 M1 A1 A1cao [4]	correct integral and limits $\left[2x - k\cos\frac{1}{2}x\right]$ where k is positive $k = 2$ answers rounding to 8.3	soi from subsequent work, condone no dx but not 180 Unsupported correct answers score 1st M1 only.

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

14 – 19 Qualifications (General)
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored

4753/01: Methods for Advanced Mathematics (C3) (Written Examination)

General Comments

There was the usual wide range of responses to this paper, with some excellent scripts in the high 60s with very well-presented solutions, to scripts which failed to achieve 20 marks. In general, there were lots of opportunities for all suitably prepared candidates to show what they could do, and only one part question – the proof in 7(ii) – which failed to attract plenty of fully correct answers.

In general, questions on calculus are well answered, showing sound knowledge of product, quotient and chain rules and integration by parts and substitution. Questions involving algebra with e^x and ln x, the modulus function, and the language of functions are perhaps less well done.

Candidates seemed to have sufficient time to answer all the questions. It is perhaps worth pointing out that when they offer a number of different solutions, it is the last attempt which is marked, not the best! So, if offering more than one solution, they should decide which is their best attempt, and cross out others. It is also worth emphasizing that when they are required to 'show' a given result on the paper, they should make sure that all the relevant steps in the argument are included.

Scripts are now scanned in for marking. It aids the overall legibility of scanned-in work if candidates do not write over partially erased pencil, or over-write existing work, and to be especially careful when writing negative signs.

Comments on Individual Questions

1 Most attempted this by considering $\pm(2x - 1) = \pm x$, some thinking that this led to four different possibilities, and indeed finding more than two solutions by faulty algebra. A few squared both sides, found the correct quadratic, and solved this by factorising or formula. Some had no idea how to start and tried to manipulate the equations with modulus signs, often ending with the answers like $|x| = 1$ or $x = \pm 1$. Others thought the modulus signs implied inequalities and either replaced them with these or introduced them into their answers.

2 Most candidates managed to get 1 mark out of 3 by writing the composite function correctly as $e^{2\ln x}$. This was often simplified by cancelling the e with the ln to get $2x$. Occasionally, they formed the composite function in the wrong order, viz 2 ln e^x, simplifying this to $2x$. Another error was to write gf(x) = g(x) f(x) = e^x.2ln x, though this was relatively rare.

3 (i) This was well answered, with the majority using the quotient rule correctly, though a significant number failed to cancel the common factor of x. The product rule works equally well for this question, and was seen occasionally.

(ii) The key to this integration by parts was selecting $u = \ln x$ and $dv/dx = 1/x^2$. Other choices gained no marks. There was evidence of 'stockpiling' negative signs in the subsequent parts formula – students could be encouraged to resolve these as they progress through the working. As the answer was given, we needed to see $\int v\, u' dx$ as $\int 1/x^2 dx$ before integrating this to get $-1/x$.

4 (i) Most students got either full marks, or gained one mark out of three for $a - b = 0.5$, thereafter making no further progress, as they failed to recognise that 'long-term' meant $e^{-kt} \to 0$, to get $a = 10.5$.

(ii) Generous follow-through was allowed on incorrect (even manufactured) values of *a* and *b*, and candidates usually showed good understanding of re-arranging the equation and taking lns of both sides to get $k = -\ln[(a - 6)/b] / 8$. Occasionally, though, their values led to the ln of a negative number, and they lost the second mark by fiddling this. Not all rounded to 2 decimal places correctly, and 0.01 after correct working was not uncommon.

5 Most candidates handled the product and chain rules well and gained three marks for a correct expression. Occasionally the multiplication by 4 was missing in the derivative of $(1 + 4x)^{1/2}$. In general, more aimed to find a common denominator than to factorise when simplifying: plenty of good work was seen. The most common reasons for loss of marks were:

- letting the 2 slide into the denominator: $x^2.2(1 + 4x)^{-1/2}$ becoming $x^2/ [2 (1+4x)^{1/2}]$;
- insufficient detail/evidence in the final stages.

6 (i) Failure to express sin(π/3) and cos (π/6) in exact form led to this mark being lost frequently: simply writing $\sin(\pi/3) + \cos(\pi/6) = \sqrt{3}$ was not sufficient.

(ii) Generally, the implicit differentiation of $\sin 2x + \cos y$ was handled well. Errors included:

- omitting the '2' in $2\cos 2x$ term,
- sign errors in the derivatives of sine and cosine,
- starting $dydx = 2 \cos 2x - \sin y dy/dx$ (= 0), and incorporating the superfluous dy/dx into the subsequent equation,
- faulty differentiation of the constant $\sqrt{3}$.

The calculation of the gradient at (π/6, π/6) by substituting for *x* and *y* in their derivative was well done, but awarded only if they made a reasonable attempt at the implicit differentiation.

7 (i) Most handled this expansion correctly, though $3^n \times 3^n = 9^{2n}$ was not uncommon.

(ii) Correct solutions to this question were the preserve of A* candidates. Most attempted it by verifying the result for various values of *n* (some quoting 'proof by exhaustion' after checking $n = 1$ to 9). Some indeed used negative numbers hoping to prove it *only* worked for positive. A popular argument was that as 3^{2n} is a multiple of 9, $3^{2n} - 1$ must therefore be a multiple of 8. Those considering the two factors made better progress, some getting as far as even x even, but failing to spot that one of two consecutive even numbers must be a multiple of 4. [The neatest solutions are using the binomial expansion of $(8 + 1)^{2n}$ and induction. Both of these were seen, and were highly commendable.]

8 (i) This was well done, with virtually all candidates being familiar with how to show that f(-*x*) = f(*x*), and the symmetry of the even function in the *y*-axis, though this property might on occasions have been expressed more accurately ('It reflects in the y-axis' being a typical example).

(ii) Of those candidates who did not start by saying f(*x*) = $1/e^x + 1/e^{-x}$ + ½, the rest divided equally between using a chain or quotient rule. A rather surprising minority multiplied e^x by $–e^{–x}$ to obtain the derivative of the bracket. A lack of brackets and/or careless positioning of minus signs led to some marks being lost, and some forgot the –2 power in their answer when using the chain rule. A few thought that f'(*x*) was the inverse rather than the derivative.

(iii) This little piece of algebra was often well done, provided that candidates did not start with f(x) =$1/e^x + 1/e^{-x}$ + ½. They should be discouraged from starting with the given result, cross-multiplying it and then working towards 1 = 1. There were some errors in expanding the brackets, or multiplying out powers of e, e.g. e^x x $e^{–x} = e^{–2x}$.

(iv) This question was not quite so well answered. It is helpful if candidates start by writing down, in terms of *x*, the integral they are trying to work out. The integration by substitution to get $\int 1/u^2\, du$ suffered from a number of errors: some just replaced d*x* by d*u* (or, omitted both d*x* and d*u* entirely, with the same result). Others moved non-constant terms to the front of the integral, or left a mixture of *u*'s and *x*'s. The integration of $1/u^2$ then caught out others, with answers such as $\ln(u^2)$ or $u^{–3}/(–3)$. It was not uncommon to see the lower limit for *u* as 1, and a few candidates changed back to *x* but substituted limits for *u*. Finally, there were numerical answers, and exact answers but with e^1 not evaluated as e.

(v) Virtually all candidates scored the first mark for equating the two functions. Thereafter, sorting out the equation to get a quadratic in e^x was done with varying degrees of success. Some took lns incorrectly; many failed to divide through by e^x, and ended up with a cubic in e^x. A number of those who got as far as $(e^x + 1)^2 = 4$ failed to consider $e^x + 1 = –2$. Some arrived at (0, ¼) fortuitously, notwithstanding algebraic errors. This failed to secure any marks.

9 (i) Candidates were split between algebraic approaches, using two of the three points to find *a* and *b*, using transformation of the *y* = sin *x* graph, and a mixture of the two. However, as the values of *a* and *b* are given, we were fairly strict in requiring evidence. Thus, simply pointing out that the *y*-intercept was 2, so *a* = 2, was not enough, especially as a number of candidates quoted '*y* = *mx* + *c*' as their justification for this. If using a stretch to show that *b* = ½ , a scale factor of 2, not ½ , as well as the direction of the stretch was required. Verifying that *a* = 2 and *b* = ½ at points A and B was not quite enough to secure all three marks.

(ii) The gradient at (0,2) was generally well done, with most candidates scoring the first 3 marks. However, good justifications of this being a maximum were less common. Many candidates simply looked at the gradient at specific values of x either side of 0 rather than using the fact that cos (*x*/2) has a greatest value of 1, or using the second derivative.

(iii) The majority had no difficulty finding $f^{-1}(x)$, though some mistakes occurred in re-arranging the equation, e.g. $x - 2 = \sin y/2 \Rightarrow y = \arcsin [2(x - 2)]$. Precision in specifying the domain and range, with $\leq$ rather than $<$ signs, and correct use of x and y or $f^{-1}(x)$ was required. This cost some candidates marks. The gradient of the inverse function was done with varying success, with -2 or ½ as common errors.

(iv) The first three marks were easy for many, but there was plenty to be done for the last mark, which was frequently lost because of poor handling of the lower limit, zero. Where other marks were lost it was usually for poor integration of sin (x/2) or, surprisingly frequently, forgetting the integral of the constant 2.

GCE Mathematics (MEI)

			Max Mark	a	b	c	d	e	u
4751/01	(C1) MEI Introduction to Advanced Mathematics	Raw	72	55	49	43	37	32	0
		UMS	100	80	70	60	50	40	0
4752/01	(C2) MEI Concepts for Advanced Mathematics	Raw	72	53	46	39	33	27	0
		UMS	100	80	70	60	50	40	0
4753/01	(C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	54	48	42	36	29	0
4753/02	(C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4753/82	(C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4753	(C3) MEI Methods for Advanced Mathematics with Coursework	UMS	100	80	70	60	50	40	0
4754/01	(C4) MEI Applications of Advanced Mathematics	Raw	90	63	56	50	44	38	0
		UMS	100	80	70	60	50	40	0
4755/01	(FP1) MEI Further Concepts for Advanced Mathematics	Raw	72	59	52	45	39	33	0
		UMS	100	80	70	60	50	40	0
4756/01	(FP2) MEI Further Methods for Advanced Mathematics	Raw	72	55	48	41	34	27	0
		UMS	100	80	70	60	50	40	0
4757/01	(FP3) MEI Further Applications of Advanced Mathematics	Raw	72	55	48	42	36	30	0
		UMS	100	80	70	60	50	40	0
4758/01	(DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	63	57	51	45	39	0
4758/02	(DE) MEI Differential Equations with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4758/82	(DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4758	(DE) MEI Differential Equations with Coursework	UMS	100	80	70	60	50	40	0
4761/01	(M1) MEI Mechanics 1	Raw	72	60	52	44	36	28	0
		UMS	100	80	70	60	50	40	0
4762/01	(M2) MEI Mechanics 2	Raw	72	64	57	51	45	39	0
		UMS	100	80	70	60	50	40	0
4763/01	(M3) MEI Mechanics 3	Raw	72	59	51	43	35	27	0
		UMS	100	80	70	60	50	40	0
4764/01	(M4) MEI Mechanics 4	Raw	72	54	47	40	33	26	0
		UMS	100	80	70	60	50	40	0
4766/01	(S1) MEI Statistics 1	Raw	72	53	45	38	31	24	0
		UMS	100	80	70	60	50	40	0
4767/01	(S2) MEI Statistics 2	Raw	72	60	53	46	39	33	0
		UMS	100	80	70	60	50	40	0
4768/01	(S3) MEI Statistics 3	Raw	72	56	49	42	35	28	0
		UMS	100	80	70	60	50	40	0
4769/01	(S4) MEI Statistics 4	Raw	72	56	49	42	35	28	0
		UMS	100	80	70	60	50	40	0
4771/01	(D1) MEI Decision Mathematics 1	Raw	72	51	45	39	33	27	0
		UMS	100	80	70	60	50	40	0
4772/01	(D2) MEI Decision Mathematics 2	Raw	72	58	53	48	43	39	0
		UMS	100	80	70	60	50	40	0
4773/01	(DC) MEI Decision Mathematics Computation	Raw	72	46	40	34	29	24	0
		UMS	100	80	70	60	50	40	0
4776/01	(NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	62	55	49	43	36	0
4776/02	(NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	0
4776/82	(NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
4776	(NM) MEI Numerical Methods with Coursework	UMS	100	80	70	60	50	40	0
4777/01	(NC) MEI Numerical Computation	Raw	72	55	47	39	32	25	0
		UMS	100	80	70	60	50	40	0